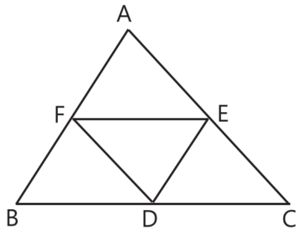
Quadrilaterals

Case Study Based Questions

Case Study 1

A metal marker has a triangular shaped metal. He welded another triangle on the mid-points of that metal, such that it appears like the following figure:



In the above figure, D, E and F are the mid-points of BC, AC and AB.

On the basis of the above information, solve the following questions:

Q 1. DE is equal to:

a. AF b. $\frac{1}{2}$AB c. BF d. All of these

Q 2. If FE = FD, then which of the following relation is correct:

a. AC = AB b. $\angle$ FED = $\angle$ ECD
c. BC = AC d. $\angle$ CAB = $\angle$ AFD

Q 3. Which type of quadrilateral BDEF?

a. Parallelogram b. Square
c. rectangle d. Trapezium

Q 4. Identify the correct relation:

a. FD $= \frac{1}{2}$ AB b. AE + FD = AC
c. AB – DE = AC d. None of these

Q 5. The sum of adjacent angles in a parallelogram is:

a. 90° b. 145°
c. 180° d. None of these

Solutions

1. (d) $\because$ D and E are the mid-points of side BC and AC.

$\therefore$ By mid-point theorem, $DE = \frac{1}{2}$ AB

But $\frac{1}{2}AB = AF = BF$

$\therefore$ $DE = \frac{1}{2}AB = AF = BF$

So, option (d) is correct.

2. (c) $FD = \frac{1}{2}AC$ [By mid-point theorem]

and $FE = \frac{1}{2}BC$

Given, FD = FE

$\therefore$ $\frac{1}{2}AC = \frac{1}{2}BC$

$AC = BC$

So, option (c) is correct.

3. (a) In quadrilateral BDEF; FE = BD and DE = BF, so quadrilateral is a parallelogram.

So, option (a) is correct.

4. (b) $\because$ $FD = \frac{1}{2}AC$ [By mid-point theorem] ...(1)

$\because$ E is the mid-point of AC.

$\therefore$ $AE = \frac{1}{2}AC$ ...(2)

Adding eqs. (1) and (2), we get

$AE + FD = AC,$

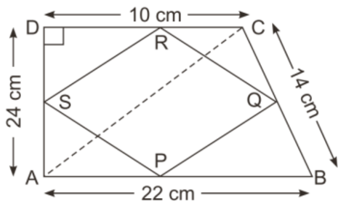
So, option (b) is correct.

5. (c) The sum of pair of adjacent angles in a parallelogram is 180°.

So, option (c) is correct.

Case Study 2

Person A has a quadrilateral shaped paper which he cut from a circular paper. Person B joined the mid-points of all sides and another quadrilateral was formed.



Above figure shows how the paper appears, side AB = 22 cm, BC = 14 cm, CD = 10 cm and AD = 24 cm.

On the basis of the above information, solve the following questions:

Q1. The measure of diagonal AC is:

a. 13 cm   b. 30 cm   c. 28 cm   d. 26 cm

Q2. If PQ || AC, then the measure of PQ is:

a. 15 cm   b. 13 cm   c. 17 cm   d. 19 cm

Q3. Quadrilateral PQRS is which of type quadrilateral?

a. Rhombus   b. Rectangle

c. Parallelogram   d. Trapezium

Q4. While proving quadrilateral is a rectangle, choose the correct option:

a. by showing opposite sides equal and each adjacent angle is 90°

b. by proving diagonals are equal

c. by proving all angles 90°

d. by proving all of the above

Q5. In any quadrilateral, the sum of all angles is:

a. 250°   b. 360°   c. 290°   d. 270°

Solutions

1. (d) $\because$ Δ ADC is a right angled triangle.

$\therefore$ $AC^2 = AD^2 + DC^2$ [Use Pythagoras theorem]

$AC^2 = 24^2 + 10^2$

$AC^2 = 576 + 100$

$AC^2 = 676$

$AC = \sqrt{676}$

$AC = 26$ cm

So, option (d) is correct.

2. (b) In ΔABC, P and Q are the mid-points of side AB and BC.

$\therefore$ $PQ = \frac{1}{2} AC$ [By mid-point theorem]

$PQ = \frac{1}{2} \times 26$

$PQ = 13$ cm

So, option (b) is correct.

3. (c) Since, P, Q, R and S are the mid-point of the sides AB, BC, CD and DA. Therefore, joining adjacent mid-point forms a parallelogram.

So, option (c) is correct.

4. (d) by proving all of the above

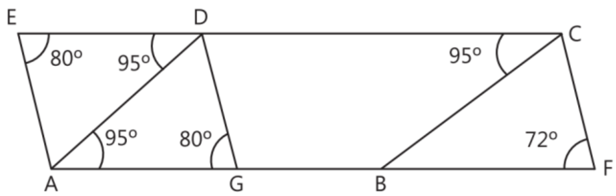
So, option (d) is correct.

5. (b) The sum of all angles in a quadrilateral is 360°.

So, option (b) is correct.

Case Study 3

A parallelogram shape park ABCD is in the middle of the city. Municipality decided to increase its area, so at the left side of park a triangle AED was added and on the right-side triangle BFC was added. At point G on AB, municipality put a swing.



On the basis of the above information, solve the following questions:

Q 1. Prove that AGDE is a parallelogram when ED || AG, AE || DG.

Q 2. Find the value of ∠ADC.

Q 3. Find the value of ∠DCF.

Solutions

1. Given, AE || DG and ED || AG

$\angle EDA = \angle DAG = 95°$ and $\angle AGD = 80°$

In Δ AED and Δ DGA,

$\angle DGA = \angle AED$ [Each 80°]

$\angle EDA = \angle DAG$ [Each 95°]

$AD = DA$ [Common]

$\therefore \Delta AED \cong \Delta DGA$ (by AAS congruence rule)

$\therefore AE = DG$ [by CPCT]

$ED = AG$ [by CPCT]

If in a quadrilateral each opposite sides are equal, then the quadrilateral is a parallelogram.

So, AGDE is a parallelogram. **Hence proved**

2. In parallelogram ABCD, $\angle DAG + \angle ADC + \angle DCB + \angle ABC = 360°$ [Sum of angles of a parallelogram]

$95° + \angle ADC + 95° + \angle ABC = 360°$

$2\angle ADC + 190° = 360°$

[$\because \angle ADC = \angle ABC$]

$2\angle ADC = 360° - 190°$

$2\angle ADC = 170°$

$\angle ADC = 85°$

3. In quadrilateral AFCD,

$\angle DAG + \angle ADC + \angle AFC + \angle DCF = 360°$

[Angle sum property of a quadrilateral]

$95° + 85° + 72° + \angle DCF = 360°$

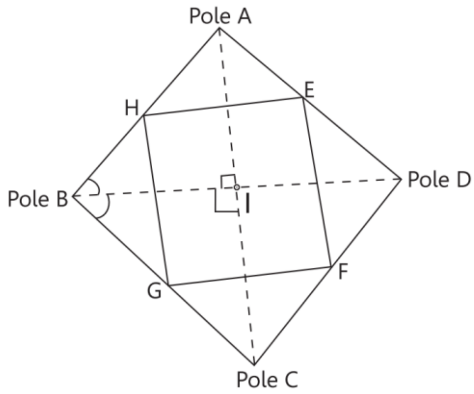
$\angle DCF = 360° - 95° - 85° - 72°$

$\angle DCF = 108°$

Case Study 4

Due to frequent robberies in the colony during night. The secretary with the members together decides to attach more lights besides the street light set by municipality. There are poles on which lights are attached.

These 4 poles are connected to each other through wire and they form a quadrilateral. Light from pole B focus light on mid-point G of wire between pole C and B, from pole C focus light on mid-point F of wire between pole C and pole D. Similarly pole D and pole A focus light on the mid-point E and H respectively.



On the basis of the above information, solve the following questions:

Q 1. If BD is the bisector of $\angle B$ then prove that I is the mid-point of AC.

Q 2. Prove that quadrilateral EFGH is a parallelogram.

Q 3. Is it true that every parallelogram is a rectangle?

Solutions

1. In ΔBIA and ΔBIC,

	$\angle ABI = \angle CBI$	[$\because$ BD is the bisector of $\angle B$]
	$BI = BI$	[Common]
	$\angle BIA = \angle BIC$	[Each 90°]
$\therefore$	$\Delta BIA \cong \Delta BIC$	[SAS congruence rule]
$\therefore$	$AI = CI$	[CPCT]

It means I is the mid-point of AC. **Hence proved**

2. Here, $HG = \frac{1}{2}AC$ [By mid-point theorem]

and $EF = \frac{1}{2}AC$ [By mid-point theorem]

$GH \parallel EF$ and $HG = EF$

If in a quadrilateral opposite side is parallel and equal then the quadrilateral is a parallelogram.

So, quadrilateral EFGH is a parallelogram.

3. It is false, because every parallelogram is not a rectangle.